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COMMENT

On Monte Carlo generation of lattice trees

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Abstract. Monte Carlo studies of site trees on the square lattice at fixed size are used for estimation of the critical exponent for the radius of gyration and the average site perimeter-to-size ratio. The latter is estimated as 1.300 ± 0.007 .

The study of lattice trees is by now a well established alternative to the analysis of lattice animals for the determination of critical exponents, since it has been known for some time that they share the same universality class (Lubensky and Isaacson 1979, Seitz and Klein 1981, Duarte and Ruskin 1981) and Monte Carlo methods were used to back this equivalence from early studies of Seitz and Klein. These authors have used a reptation algorithm that differed from existing studies of animals (Gould and Holl 1981), relying on the permutation of cluster sites and perimeter sites to generate animal configurations. Recently, Glaus (1985) presented results from a Monte Carlo procedure which generates trees at variable size value and thus managed to obtain the critical fugacity μ and the exponent θ for the generating function of the total number of trees, N_s :

$$N_s \propto \mu^s s^{-\theta} \quad \text{for } s \rightarrow \infty \quad (1)$$

in better agreement with the Parisi-Sourlas prediction than the series studies (Duarte and Ruskin 1981, Gaunt *et al* 1982) most noticeably in three dimensions.

We report here on the results of a Monte Carlo generation at fixed size of square site trees. The algorithm starts from an initial tree configuration: a random choice is made from all its sites with only one neighbour (its endpoints) and the chosen site is deleted. The site perimeter set is then determined and a perimeter chosen at random. It will be added to it if it leads to a tree (i.e. if occupation would transform it into a new endpoint). If this condition fails and a loop is formed the Monte Carlo procedure returns to the initial configuration and the process begins again.

We have calculated the radius of gyration and the average perimeter-to-size ratio for various values of s up to $s = 700$: both quantities can be calculated exactly up to size 17 (see Family *et al* (1985) for the radius of gyration). Our values already agree with the perimeter-to-size ratio values within 0.5%, for $s = 3, 4, 5$, with the deviation declining slowly with size.

Larger values, calculated in tables 1 and 2 from the exact series expansion of Duarte (1986), follow a pattern familiar from the analyses of similar averages for lattice animals (see, e.g., Duarte (1985) or Duarte and Ruskin (1982)). Extrapolation through Neville

Table 1. Listing of average perimeter-to-size ratios for trees on the square lattice.

Ratio	
4	2.1667
5	2.0073
6	1.9042
7	1.8266
8	1.7655
9	1.7172
10	1.6784
11	1.6462
12	1.6191
13	1.5959
14	1.5758
15	1.5584
16	1.5430
17	1.5293

Table 2. Listing of central averages for the radius of gyration of square site trees. Values marked with * refer to 1.5 million steps averages not yet in equilibrium (earlier values are based on runs of, typically, $\frac{1}{2}$ -5 million steps).

Radius	
11	1.9646
15	2.4416
17	2.6604
21	3.07
31	3.98
61	6.22
81	7.49
121	9.76
201	13.55*
381	21.3*

tables compounded with fittings lead to a limiting ratio estimate of

$$\langle t/s \rangle = 1.300 \pm 0.007 \quad (2)$$

with t the perimeter in the percolation sense. From our Monte Carlo data we found this ratio to be about 1.30 ± 0.02 , compatible with this series estimate. Both estimates are definitely higher than the ratio for animals with loops (Duarte and Ruskin 1982).

For the radius of gyration, Glaus (1985) gives $\nu = 0.640$ (8) and Seitz and Klein quoted 0.615, both for trees. These values should be set against the phenomenological renormalisation prediction of $\nu = 0.6406$ (1) (Kertész 1986, with earlier references) for animals, usually taken as a benchmark for Monte Carlo and series work (e.g. for Gould and Holl (1981), $\nu = 0.65$).

Our results are given in figure 1, where the successive slopes of the log-log plot are represented as a function of the inverse size. The values for $s > 200$ are affected by longer relaxation times. The overall estimate from the intermediate sizes is

$$\nu = 0.650 \pm 0.015 \quad (3)$$

which seems to compare favourably with the current Monte Carlo estimates with fixed size. The generation of trees rather than animals avoids the complex check for

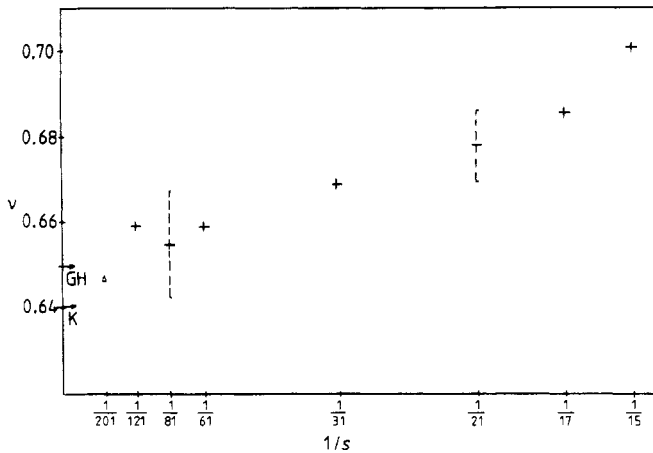


Figure 1. Plot of the log-log successive slopes (marked +) of the radius of gyration against size as a function of $1/s$. The error bars for sizes 21 and 81 are indicated on the figure as well as the estimates of Kertész (K) and Gould and Holl (GH), marked by arrows on the vertical axis. A Δ denotes the $s = 201$ -121 value, not yet in equilibrium.

connectivity after site removal needed in the simulation of lattice animals (Gould and Holl 1981, Peters *et al* 1979): this is probably its greatest advantage. The main drawback, common to reptation algorithms, resides in very long relaxation times. It seems possible therefore to use a fixed size simulation as an alternative to the Glaus (1985) use of a grand canonical ensemble for the determination of the exponent ν for lattice animals by Monte Carlo methods.

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